Two identical looking vases sit on a table.
Each vase is filled with ten marbles: red and green.
You know the ratio of red to green in each vase.
Your friend hands you a vase. You don't know which one.
You randomly pick a marble from the vase.
You replace the marble and randomly pick another.
You replace and sample again.
Can you figure out which vase you hold?

We can answer this with Bayes' Rule.

$$Pr(A/R) = Pr(A)\,Pr(R/A) / Pr(A)\,Pr(R/A) + Pr(B)\,Pr(R/B)$$

We know the probability of getting each color marble given each vase:

Pr(R/A) = Number of Red Marbles in Vase A / Total Marbles in Vase A

Pr(G/A) = Number of Green Marbles in Vase A / Total Marbles in Vase A

Pr(R/B) = Number of Red Marbles in Vase B / Total Marbles in Vase B

Pr(G/B) = Number of Green Marbles in Vase B / Total Marbles in Vase B

We know the probability of getting Vase A:

Pr(A) = 1/2

We know the probability of getting Vase B:

Pr(B) = 1/2

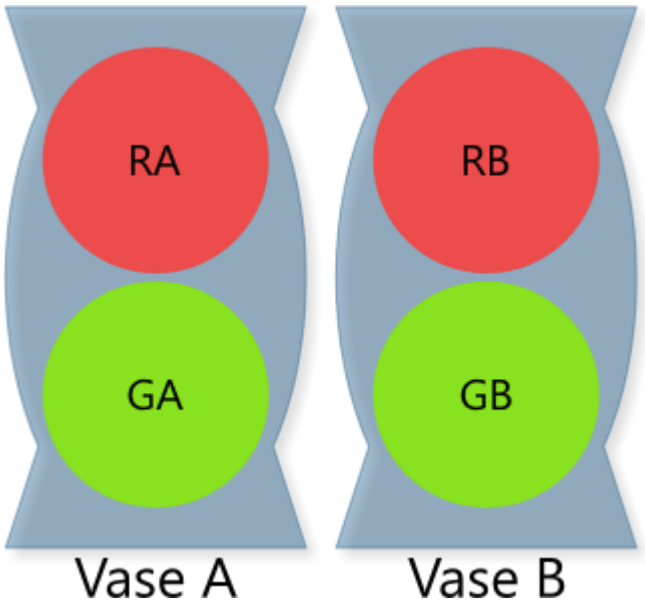
We know the total probability of getting a red marble:

Pr(R) = Pr(R/A) P(A) + Pr(R/B) Pr(B)

We know the total probability of getting a green marble:

Pr(G) = Pr(G/A) Pr(A) + Pr(G/B) Pr(B)

Now, we want to work the other way. Given a certain sample, what is the probability that it came from Vase A, and what is the probability that it came from Vase B. Below is a diagram illustrating the four possible outcomes of the first sample.



As we can see from the diagram, the probability that a red marble came from Vase A is:

RA / RA + RB or as Bayes' Rule: Pr(A/R) = Pr(A) Pr(R/A) / Pr(A) Pr(R/A) + Pr(B) Pr(R/B)

The probability that it came from Vase B is:

RB / RB + RA or as Bayes' Rule: Pr(B/R) = Pr(B) Pr(R/B) / Pr(B) Pr(R/B) + Pr(A) Pr(R/A)

If it is a green marble, the probability that it came from Vase A is:

GA / GA + GB or as Bayes' Rule: Pr(A/G) = Pr(A) Pr(G/A) / Pr(A) Pr(G/A) + Pr(B) Pr(G/B)

The probability that it came from Vase B is:

GB / GA + GB or as Bayes' Rule: Pr(B/G) = Pr(B) Pr(G/B) / Pr(B) Pr(G/B) + Pr(A) Pr(G/A)

So, what about a sample that looks like: RGGRRGGG ?

The probability that it came from Vase A would be:

P(A/RGGRRGGG) = Pr(A) Pr(RGGRRGGG/A) / Pr(A) Pr(RGGRRGGG/A) + Pr(B) Pr(RGGRRGGG/B)

The probability that it came from Vase B would be:

P(B/RGGRRGGG) = Pr(B) Pr(RGGRRGGG/B) / Pr(A) Pr(RGRRGGG/A) + Pr(B) Pr(RGGRRGGG/B)

How do we figure the probability of RGGRRGGG given Vase A?

Pr(RGGRRGGG/A) = Pr(R/A) * Pr(G/A) * Pr(G/A) * Pr(R/A) * Pr(R/A) * Pr(G/A) * Pr(G/A) * Pr(G/A)

How do we figure the probability of RGGRRGGG given Vase B?

Pr(RGGRRGGG/B) = Pr(R/B) * Pr(G/B) * Pr(G/B) * Pr(R/B) * Pr(R/B) * Pr(G/B) * Pr(G/B) * Pr(G/B)